GMACS SPECTROGRAPH PARAMETERS

Table 1 describes a spectrograph with a specific set of functional requirements. In this section we derive key parameters of a spectrograph on the GMT with these requirements and show that the instrument must have optics (collimator focal length, camera FOV, etc.) that are tightly constrained. Choices must be made, of course, but we can show, for example, that the spectrograph collimator must have a focal length of 1800-2600mm, based on reasonable assumptions.

Table 1: Summary of principal functional requirements and goals.

Parameter	**Requirement / Goal**	**Comments**
Field of View	30 arcmin sq. / 50 arcmin sq.	
Wavelength coverage	370-950nm / 340-1000nm	
Spectral Resolution	Blue: 1000-6000 Red: 1000-6000	0.7” slit width, full coverage at lower resolutions, wavelength coverage at higher resolutions is sacrificed
Image Quality	0.30 / 0.15 arcsec	80% EE
Spectral Stability	0.3 / 0.1 spectral resolution elements/hour	
Grating Exchange	2 / ≥2 resolutions	Multiple wavelength regions
Slit Mask Exchange	20 / ≥20	Cassette style mask changer

Assumptions & Constraints

As discussed above, the GMACS science objectives identify the highest priorities for the instrument as a ~5.5 arcmin long slit and wide wavelength coverage at resolution ~2100 (which we take to mean in the middle of any given band). The "red" wavelength range was the consensus highest priority. These objectives translate to a simultaneous coverage of roughly 605nm to 955nm and a dispersion of ~0.36nm in a single exposure. There is substantial interest in a higher resolution capability; roughly a resolution of ~6000 (dispersion of ~0.13nm) is desired. The dispersion is when using a 0.7 arcsec wide slit. We will assume these objectives below.

We further assume a fiducial telescope focal length of 207,588.8mm and an aperture of 25,448mm (*f*/8.16), as given by the "official" GMT Zemax design.

Derivation of Spectrograph Parameters

Number of Pixels in the non-dispersion direction

At fixed telescope focal length and slit length, the required detector size (along the slit) is a function of the ratio of the camera and collimator focal lengths:

$$number\ of\ pixels\ along\ slit = \frac{slit\ length\ (arcseconds)\,\frac{f_{telescope}}{206265}\frac{f_{cam}}{f_{coll}}}{pixels\ size}$$

where all units are in mm. For the fiducial GMT, detectors with 15μm pixels, and a 5.5 arcmin long slit, this simplifies to

$$number\ of\ pixels\ along\ slit = 22141\frac{f_{cam}}{f_{coll}}$$

which of course suggests that the smaller the ratio of the two focal lengths, the fewer pixels are needed to satisfy the slit length requirement. However, by Liouville's Theorem the etendue of the system is conserved (since we plan to use normal lenses and mirrors), so the *f*/# of the collimator must be the same as the *f*/# of the telescope and the *f*/# of the camera must be

$$Camera\ f/\# = \frac{f_{cam}}{f_{coll}}(Telescope\ f/\#)$$

Since optics faster than ~*f*/1 are impractical, the number of pixels along the slit must therefore be more than ~2713 for a 5.5 arcmin long slit (which roughly corresponds to the minimum required field-of-view). Obviously, a longer slit (and greater field-of-view) requires more pixels: the GMACS field-of-view goal corresponds to ~3500 pixels.

Optical constraints

Another key constraint is the camera field-of-view. According to Bernstein (several talks), camera FOV must be less than ~22° (0.38^R); for some families of optical design the maximum FOV can be significantly smaller (e.g. Schmidt-like designs seem limited to FOV less than ~15°). The rough FOV of a camera is

$$FOV_{cam} \approx \frac{Physical\ size\ of\ detector\ array}{f_{cam}}$$

(Note: this assume relatively small angles and that the camera is located very close to the pupil/grating). For a detector with an aspect ratio of *N* (i.e. there are *N* times more pixels along the dispersion direction than along the slit direction) on the fiducial GMT and 15μm pixels the camera FOV is roughly

$$FOV_{cam} = \frac{332(N^2+1)^{1/2}}{f_{coll}}$$

where we have used the relations given above to recast the camera focal length in terms of the number of pixels required to achieve a 5.5 arcmin slit length. Given the maximum practical FOV for any family of potential designs, this suggests a minimum possible collimator focal length for any potential detector array. For example, if we suppose that Schmidt-like designs are not practical for FOV larger than ~15° (0.26^R) and we are constrained to use square-format detectors, then the collimator must have a focal length greater than ~1640mm. This of course suggests a camera focal length greater than ~200mm as well.

More generally, detectors with other formats are available, although aspect ratios of greater than 4:1 are uncommon. Assuming the camera FOV must be kept below about 15° or 22°, Table 2 shows the minimum required collimator focal lengths (again assuming the GMT fiducial, 15μm pixels, and a 5.5 arcmin long slit).

Table 2: Rough minimum collimator focal lengths (mm) for various detector aspect ratios

Aspect ratio	Camera FOV < 15°	Camera FOV < 22°
1:1	1805	1236
2:1	2855	1954
3:1	4037	2763
4:1	5264	3602

Note that the estimates in Table 2 assume a 5.5 arcmin long slit, which corresponds to the minimum requirement for GMACS field-of-view. The goal will require a longer slit and, therefore, longer minimum collimator focal length.

The collimator focal length and camera focal length also determine the dispersion and simultaneous wavelength coverage and other key parameters of the instrument. For example, the grating size is roughly the collimator focal length divided by the telescope *f*/# divided by the cosine of the incident angle. Since the practical size limit of VPH gratings seems to be ~450mm (Clemens *private communication*; see below) and there is a strong desire to have resolutions that suggest incident angles of ~45° (to satisfy the Bragg condition), this suggests that the collimator focal length should be less than ~2600mm. This implies a joint condition on the Camera FOV and detector aspect ratio: Schmidt-like optical designs can be used for the spectrograph camera, but only with detector aspect ratios of 1:1. We note that the availability (at moderate cost) of CCDs suggests that Schmidt-like camera designs may not be suitable; wide FOV refractive designs are likely better choices for the instrument.

Interest in a higher resolution constrains the collimator focal length as well. The resolution can be expressed as

$$Resolution = \frac{m\lambda}{slit\ width}(line\ density)f_{coll}$$

Assuming first order operation and the nominal slit width of 0.7 arcsec (0.704mm), a resolution of ~6000, and maximum line density of a VPH grating of ~3000 l/mm (Clemens *private communication*), this suggests that the collimator should have a focal length of more than ~1834mm.

Constraints on width of detector

The simultaneous wavelength coverage requirement also imposes several constraints. For example, for the red channel to cover a range of 605nm to 955nm at a dispersion of ~0.36nm, there must be at least 972 resolution elements across the detector array. The nominal slit (and hence roughly a single resolution element) projects onto (15μm) pixels

$$number\ of\ pixels\ in\ 0.7\ \mathrm{arcsec}\ slit = 46.97\frac{f_{cam}}{f_{coll}}$$

Since the ratio of the camera and collimator focal lengths must be greater than 0.122 (so that the camera is slower than *f*/1) and we require at least 972 resolution elements, this implies that the size of the detector in the red channel should be larger than ~5595 to formally achieve the stated objective. A small change in the resolution goal to ~2380 would allow use of 6144 pixel wide detectors (at short camera focal length).

Possible combinations

Table 3 shows some combinations of collimator and camera focal lengths and FOV. Some CCD widths were assumed and the resolution (at 780nm) for full 605nm to 955nm coverage is shown; the orthogonal dimension is given so as to cover a 5.5 arcmin slit (the number of pixels required for a 5.5 arcmin slit is shown). For each combination a detector format was assumed that gives roughly adequate resolution and coverage (although in some cases the target requirement is under-satisfied by ~25%). The approximate resolution is given in Table 3.

Detector sizes are not generally arbitrary and typically are modulo 1024. For example, CCDs with 6144, 8192 (either monolithic or as a mosaic of 4096 wide individual detectors), and 12288 (mosaic of 6144 wide individual detectors) pixels are commercially available, albeit with some restrictions on the format in the orthogonal direction (seemingly limited by the largest silicon wafer that can be processed). These are reflected in the detector formats in Table 2 only in the dispersion direction; in most cases the number of pixels along the slit is calculated and may only be available in a custom CCD (and will obviously be larger for a longer slit and instrument FOV).

Table 4 shows specific combinations of collimator and camera parameters when used to cover several detector formats. This gives a sense of the changes incumbent on the optical parameters with change in detector format.

Many of the combinations in Table 3 (and 4) are not feasible. For example, Schmidt-like camera designs that can be made as fast as *f*/1 are probably not possible, since the required FOV is too large (18°-19°). In fact, there are no combinations in Table 3 that seem amenable to a Schmidt-like approach for the camera (except possible an *f/*2.5 system with a relatively long focal length). Furthermore, many of the FOV are significantly larger than 22° (essentially all at shorter collimator focal length).

Nonetheless, it is encouraging that there is some range of potential combinations of collimator and camera designs that are acceptable. We note that many designs similar to those shown in Table 3 and Table 4 exist in other instruments for large telescopes. Indeed the optics for the previous conceptual design for GMACS are close to some of the combinations. We do not anticipate that the final optical design of this version of GMACS will be impossible. Inspection of Table 3 and guesses at possible detector format can be used to examine a few possible spectrograph configurations. Configurations for one possible two-arm spectrograph are shown in Table 5.

Table 3: Combinations of collimator and camera parameters

Collimator focal length (mm)	*f*/1 camera focal length (mm)	Camera FOV (5' long slit and R=2379; 2713×6144) (°)	*f*/1.25 camera focal length (mm)	Camera FOV (5' long slit and R=2537; 3392×8192) (°)	*f*/1.5 camera focal length (mm)	Camera FOV (5' long slit and R=2114; 4070×8192) (°)
1800	220.6	26.2	275.7	27.6	330.9	23.8
2000	245.1	23.6	306.4	24.9	367.6	21.4
2200	269.6	21.4	337.0	22.6	404.4	19.4
2300	294.1	20.5	352.3	21.6	422.8	18.6
2400	294.1	19.6	367.6	20.7	441.2	17.8
2448	300	19.2	375	20.3	450	17.5
2600	318.6	18.1	398.3	19.1	477.9	16.4

Collimator focal length (mm)	*f*/1.7 camera focal length (mm)	Camera FOV (5' long slit and R=2799; 4613×12288) (°)	*f*/1.75 camera focal length (mm)	Camera FOV (5' long slit and R=2719; 4748×12288) (°)	*f*/1.85 camera focal length (mm)	Camera FOV (5' long slit and R=2572; 5020×12288) (°)
1800	375	30.1	386.0	29.3	408.1	28.0
2000	416.7	27.1	428.9	26.4	453.4	25.2
2200	458.3	24.6	471.8	24.0	498.8	22.9
2300	479.2	23.5	493.3	23.0	521.4	21.9
2400	500	22.6	514.7	22.0	544.1	21.0
2448	510	22.1	525	21.6	555	20.6
2600	541.7	20.8	557.6	20.3	589.5	19.4

Collimator focal length (mm)	*f*/2 camera focal length (mm)	Camera FOV (5' long slit and R=2379; 5427×12288) (°)	*f*/2.25 camera focal length (mm)	Camera FOV (5' long slit and R=2114; 6105×12288) (°)	*f*/2.5 camera focal length (mm)	Camera FOV (5' long slit and R=1903; 6783×12288) (°)
1800	441.2	26.2	496.3	23.8	551.5	21.9
2000	490.2	23.6	551.5	21.4	612.7	19.7
2200	539.2	21.4	606.6	19.4	674.0	17.9
2300	563.7	20.5	634.2	18.6	704.7	17.1
2400	588.2	19.6	661.8	17.8	735.3	16.4
2448	600	19.2	675	17.5	750	16.1
2600	637.25	18.1	716.9	16.4	796.6	15.1

Table 4: Combinations of collimator and camera parameters for different CCD formats

Collimator focal length (mm)	*f*/1.7 camera focal length (mm)	Camera FOV (5.5' long slit and R=1866; 4193×8192) (°)	Camera FOV (5.5' long slit and R=2799; 4193×12288) (°)	Camera FOV (5.5' long slit and R=3731; 4193×16384) (°)
1800	375.0	21.5	30.1	39.0
2000	416.7	19.4	27.1	35.1
2200	458.3	17.6	24.6	31.9
2300	479.2	16.9	23.5	30.5
2400	500.0	16.2	22.6	29.3
2448	510.0	15.8	22.1	28.7
2600	541.7	14.9	20.8	27.0

Table 5: Example Configurations of a Two Arm Spectrograph

F_{coll}	F_{cam}	$f/\#_{cam}$	CCD format	FOV_{cam}	VPH (l/mm)	Cam-coll angle	Simultaneous wavelength coverage	Disp (nm)	Resolution @ center λ	Slit length (arcmin)
2200	600	1.7	8196×12288	17.08	1180	32.9	355-605	0.260	1844	7.4
2200	600	1.7	8196×12288	17.08	830	37.8	605-955	0.365	2137	7.4
2200	600	1.7	8196×12288	17.08	2800	88.9	461-539	0.082	6122	7.4
2200	600	1.7	8196×12288	17.08	2000	88.9	645-755	0.114	6122	7.4
2200	600	1.7	8196×12288	17.08	660	18.2	250-710	0.479	1002	7.4
2200	600	1.7	8196×12288	17.08	405	18.2	406-1155	0.781	999	7.4